



Cambridge International AS & A Level

CANDIDATE
NAME

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CENTRE
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FURTHER MATHEMATICS

9231/21

Paper 2 Further Pure Mathematics 2

October/November 2021

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages.

- 1 Find the Maclaurin's series for $e^x \tan x$ from first principles up to and including the term in x^2 . [5]

- 2 The matrix \mathbf{A} is given by

$$\mathbf{A} = \begin{pmatrix} -1 & 2 & 12 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}.$$

Use the characteristic equation of \mathbf{A} to show that

$$\mathbf{A}^4 = p\mathbf{A}^2 + q\mathbf{I},$$

where p and q are integers to be determined.

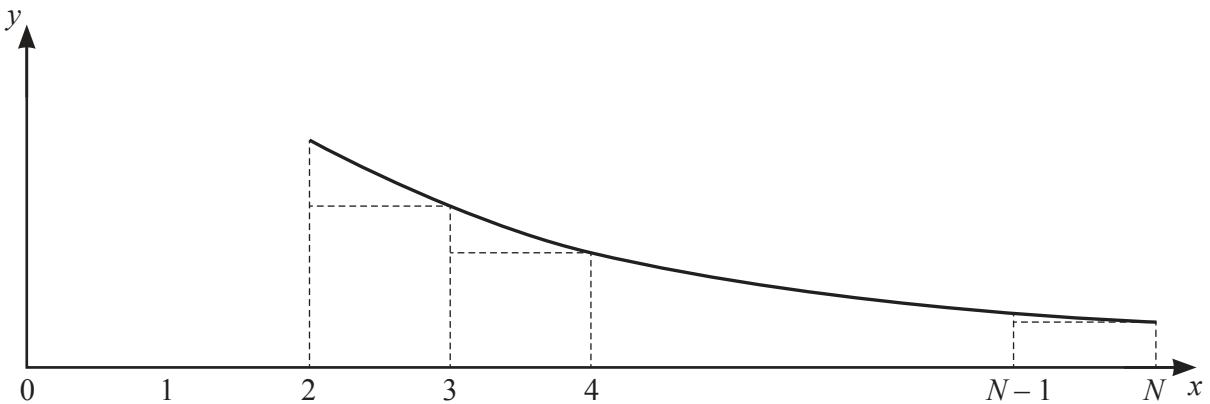
[6]

- 3** The curve C has equation

$$xy^3 - 4x^3y = 3.$$

- (a) Show that, at the point $(-1, 1)$ on C , $\frac{dy}{dx} = 11$. [3]

- (b)** Find the value of $\frac{d^2y}{dx^2}$ at the point $(-1, 1)$. [5]



The diagram shows the curve with equation $y = \frac{\ln x}{x^2}$ for $x \geq 2$, together with a set of $(N-2)$ rectangles of unit width.

- (a) By considering the sum of the areas of these rectangles, show that

$$\sum_{r=1}^N \frac{\ln r}{r^2} < \frac{2+3\ln 2}{4} - \frac{1+\ln N}{N}. \quad [7]$$

- (b)** Use a similar method to find, in terms of N , a lower bound for $\sum_{r=1}^N \frac{\ln r}{r^2}$. [3]

- 5** Find the particular solution of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4 \cos x,$$

given that, when $x = 0$, $y = -4$ and $\frac{dy}{dx} = 3$.

[11]

6 (a) Use de Moivre's theorem to show that

$$\operatorname{cosec} 5\theta = \frac{\operatorname{cosec}^5 \theta}{5 \operatorname{cosec}^4 \theta - 20 \operatorname{cosec}^2 \theta + 16}. \quad [6]$$

(b) Hence obtain the roots of the equation

$$x^5 - 10x^4 + 40x^2 - 32 = 0$$

in the form $\operatorname{cosec}(q\pi)$, where q is rational.

[4]

- 7 (a) Show that an appropriate integrating factor for

$$\sqrt{x^2 - 1} \frac{dy}{dx} + y = x^2 - x\sqrt{x^2 - 1}$$

is $x + \sqrt{x^2 - 1}$.

[4]

(b) Hence find the solution of the differential equation

$$\sqrt{x^2 - 1} \frac{dy}{dx} + y = x^2 - x\sqrt{x^2 - 1}$$

for which $y = 1$ when $x = \frac{5}{4}$. Give your answer in the form $y = f(x)$.

[7]

- 8 (a)** Starting from the definition of \cosh in terms of exponentials, prove that

$$2 \cosh^2 A = \cosh 2A + 1.$$

[3]

The curve C has parametric equations

$$x = 2 \cosh 2t + 3t, \quad y = \frac{3}{2} \cosh 2t - 4t, \quad \text{for } -\frac{1}{2} \leq t \leq \frac{1}{2}.$$

The area of the surface generated when C is rotated through 2π radians about the y -axis is denoted by A .

- (b) (i)** Show that $A = 10\pi \int_{-\frac{1}{2}}^{\frac{1}{2}} (2 \cosh 2t + 3t) \cosh 2t \, dt$.

[4]

(ii) Hence find A in terms of π and e .

[7]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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